## Penguin pollution in the $B^{0} \rightarrow J / \psi K_{S}$ decay

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Abstract: We present the most complete analysis of the penguin correction to the extraction of the standard-model parameter $\sin \left(2 \phi_{1}\right)$ from the $B^{0} \rightarrow J / \psi K_{S}$ decay up to leading power in $1 / m_{b}$ and to next-to-leading order in $\alpha_{s}, \phi_{1}$ being the weak phase, $m_{b}$ the $b$ quark mass, and $\alpha_{s}$ the strong coupling constant. The deviation $\Delta S_{J / \psi K_{S}}$ of the mixing-induced CP asymmetry from $\sin \left(2 \phi_{1}\right)$ and the direct CP asymmetry $A_{J / \psi K_{S}}$ are both found to be of $O\left(10^{-3}\right)$ in a formalism that combines the QCD-improved factorization and perturbative QCD approaches. The above results, different from those of $O\left(10^{-4}\right)$ and of $O\left(10^{-2}\right)$ obtained in the previous calculations, provide an important standard-model reference for verifying new physics from the $B^{0} \rightarrow J / \psi K_{S}$ data.

Keywords: CP violation, Standard Model, Rare Decays, B-Physics.

The $B^{0} \rightarrow J / \psi K_{S}$ decay has been regarded as the golden mode for extracting the standard-model parameter $\sin \left(2 \phi_{1}\right)$ []], $\phi_{1}$ being the weak phase of the Cabbibo-KobayashiMaskawa (CKM) matrix element $V_{\mathrm{td}}$. Though it is believed that the penguin pollution in this mode is negligible, a complete and reliable estimate of its effect is not yet available. Such an estimate is essential, especially when $B$ physics is entering the era of precision measurement. Note that the data of the mixing-induced CP asymmetries in $B$ meson decays, such as of $S_{J / \psi K_{S}}$, are not included in the CKM [2] or UT [3] fitter. Understanding the penguin effect, one will have a more concrete idea of whether a discrepancy between the measured $S_{J / \psi K_{S}}$ and $\sin \left(2 \phi_{1}\right)$ from the fitters is a signal of new physics. At the same time, the direct CP asymmetries of the $B \rightarrow J / \psi K$ decays, also related to the penguin correction, are expected to be vanishingly small. A reliable estimate of the penguin effect, when confronted with the precision measurement of $A_{\mathrm{CP}}(B \rightarrow J / \psi K)$, can reveal a new physics signal. It has been claimed that $A_{\mathrm{CP}}(B \rightarrow J / \psi K)$ observed at $1 \%$ or higher would indicate new physics definitely (4.

The previous calculation of the deviation $\Delta S_{J / \psi K_{S}} \equiv S_{J / \psi K_{S}}-\sin \left(2 \phi_{1}\right) \sim O\left(10^{-4}\right)$ [5] has taken into account the corrections to the $B-\bar{B}$ mixing [6] and to the decay amplitude. However, only the $u$-quark-loop contribution from the tree operators was included in the latter. We shall point out that the contribution from the penguin operators has been overlooked in [5], which may result in a larger deviation. The model-independent analysis in [7] gave $\Delta S_{J / \psi K_{S}}=0.000 \pm 0.012$, whose theoretical uncertainty is comparable to the current systematical error in the $S_{J / \psi K_{S}}$ measurement. This method relies on the input of the $B^{0} \rightarrow J / \psi \pi^{0}$ data in order to fix the penguin amplitude (including its strong phase). The experimental error then propagates into their uncertainty of $O\left(10^{-2}\right)$, which should have been overestimated. Though the $B^{0} \rightarrow J / \psi \pi^{0}$ data will get precise in the future, $\mathrm{SU}(3)$ symmetry breaking effects are also going to emerge, and one will lose the control of the theoretical uncertainty eventually, if using this method. Hence, we are motivated to reinvestigate this important subject by computing the complete penguin correction to the $B^{0} \rightarrow J / \psi K_{S}$ amplitude.

In this work we shall adopt a special formalism [8] that combines the QCD-improved factorization (QCDF) [6] and perturbative QCD (PQCD) [10, 11] approaches. The factorizable contribution in the $B \rightarrow J / \psi K$ decays contains the $B \rightarrow K$ form factor $F_{+}^{\mathrm{BK}}\left(m_{J / \psi}^{2}\right)$ evaluated at the invariant mass squared of the $J / \psi$ meson. Because of the heavy $m_{J / \psi}$, the energy release involved in $F_{+}^{\mathrm{BK}}\left(m_{J / \psi}^{2}\right)$ is small, and it is unlikely to further factorize it into a convolution of a hard kernel with the $B$ meson and kaon distribution amplitudes as in PQCD. Therefore, we adopt the QCDF formalism to handle the factorizable amplitude. The nonfactorizable spectator contribution is essential in the color-suppressed category of $B$ meson decays (12, 13], the reason the naive factorization assumption (14) does not apply to the $B \rightarrow J / \psi K$ decays. For this piece, QCDF is not appropriate due to the end-point singularity from vanishing parton momenta, when the twits-3 kaon distribution amplitudes are included (15). Therefore, we employ the PQCD approach based on $k_{T}$ factorization theorem, which is free of the end-point singularity. As argued in 16], the nonfactorizable contribution has a characteristic hard scale higher than that in the $B$ meson transition form factor. This mixed formalism has been applied to $B$ meson decays
into various charmonium states [8], and results for the branching ratios are consistent with the observed values.

The mixing-induced CP asymmetry $S_{J / \psi K_{S}}$ and the direct CP asymmetry $A_{J / \psi K_{S}}$ of the $B^{0} \rightarrow J / \psi K_{S}$ decay are defined by

$$
\begin{align*}
S_{J / \psi K_{S}} & =\frac{2 \operatorname{Im} \lambda_{J / \psi K_{S}}}{1+\left|\lambda_{J / \psi K_{S}}\right|^{2}} \\
A_{J / \psi K_{S}} & =\frac{\left|\lambda_{J / \psi K_{S}}\right|^{2}-1}{1+\left|\lambda_{J / \psi K_{S}}\right|^{2}} \\
\lambda_{J / \psi K_{S}} & =\frac{q}{p} \frac{\mathcal{A}\left(\bar{B}^{0} \rightarrow J / \psi K_{S}\right)}{\mathcal{A}\left(B^{0} \rightarrow J / \psi K_{S}\right)} \tag{1}
\end{align*}
$$

where the ratio $q / p$ is related to the non-diagonal elements of the mixing matrix, and $\mathcal{A}$ the decay amplitude. Note that a tiny $K_{L}$ admixture could exist in the experimentally reconstructed $K_{S}$ final state, such that CP violation in the $K-\bar{K}$ mixing contributes to $S_{J / \psi K_{S}}$ and $A_{J / \psi K_{S}}$. However, we shall concentrate only on the corrections from the $B$ meson system in this work, and refer the inclusion of the corrections from the kaon system to 17 .

We aim at an accuracy up to leading power in $1 / m_{b}$ and to next-to-leading order in $\alpha_{s}$, $m_{b}$ being the $b$ quark mass and $\alpha_{s}$ the strong coupling constant. At this level of accuracy, the correction to the mixing factor $q / p$, computed reliably in the framework of operator product expansion with the large logarithm $\alpha_{s} \ln \left(m_{W} / m_{b}\right)$ being resummed [5], gives

$$
\begin{equation*}
\Delta S_{J / \psi K_{S}}^{\operatorname{mix}}=(2.08 \pm 1.23) \times 10^{-4}, \tag{2}
\end{equation*}
$$

where $m_{W}$ is the $W$ boson mass. Note that the old value $\sin \left(2 \phi_{1}\right) \approx 0.736$ has been input to derive the above result. Using the updated one $\sin \left(2 \phi_{1}\right) \approx 0.684$ [18] changes eq. (2) by at most $10 \%$, namely, by $O\left(10^{-5}\right)$, which is absolutely negligible. The influence of the $B-\bar{B}$ mixing on the tagging leads to the direct CP asymmetry [55, 19],

$$
\begin{equation*}
A_{J / \psi K_{S}}^{\operatorname{mix}}=(2.59 \pm 1.48) \times 10^{-4} \tag{3}
\end{equation*}
$$

which is consistent with the estimate in [20]. Both eqs. (2) and (3) are of $O\left(10^{-4}\right)$.
The $B^{0} \rightarrow J / \psi K^{0}$ decay amplitude is decomposed into

$$
\begin{equation*}
\mathcal{A}\left(B^{0} \rightarrow J / \psi K^{0}\right)=V_{c b}^{*} V_{c s}\left(\mathcal{A}_{J / \psi K^{0}}^{(c)}+\mathcal{A}_{J / \psi K^{0}}^{(t)}\right)+V_{u b}^{*} V_{u s}\left(\mathcal{A}_{J / \psi K^{0}}^{(u)}+\mathcal{A}_{J / \psi K^{0}}^{(t)}\right), \tag{4}
\end{equation*}
$$

where the unitarity relation $V_{t b}^{*} V_{t s}=-V_{u b}^{*} V_{u s}-V_{c b}^{*} V_{c s}$ for the CKM matrix elements has been inserted, and the second term carries the weak phase $\phi_{3}$. The dominant tree amplitude $\mathcal{A}_{J / \psi K^{0}}^{(c)}$ is responsible for the $B^{0} \rightarrow J / \psi K^{0}$ branching ratio, and $\mathcal{A}_{J / \psi K^{0}}^{(u, t)}$ are the penguin pollution in the decay. The amplitude $\mathcal{A}_{J / \psi K^{0}}^{(u)}$, receiving the contribution from the $u$-quark loop through the BSS mechanism [21], has been estimated naively in [5], while $\mathcal{A}_{J / \psi K^{0}}^{(t)}$ from the penguin operators has been missed. We shall calculate these amplitudes in our formalism below.

The leading-power QCDF formulas for the factorizable contributions to $\mathcal{A}_{J / \psi K^{0}}^{(c, t)}$ are written as [8]

$$
\begin{align*}
& \mathcal{A}_{J / \psi K^{0}}^{(c) f}=2 \sqrt{2 N_{c}} \int_{0}^{1} d x_{3} \Psi^{L}\left(x_{3}\right) a_{2}\left(x_{3}, t\right) F_{+}^{\mathrm{BK}}\left(m_{J / \psi}^{2}\right) \\
& \mathcal{A}_{J / \psi K^{0}}^{(t) f}=2 \sqrt{2 N_{c}} \int_{0}^{1} d x_{3} \Psi^{L}\left(x_{3}\right)\left[a_{3}\left(x_{3}, t\right)+a_{5}\left(x_{3}, t\right)\right] F_{+}^{\mathrm{BK}}\left(m_{J / \psi}^{2}\right) \tag{5}
\end{align*}
$$

with $N_{c}=3$ being the number of colors, $\Psi^{L}$ the twist- $2 J / \psi$ meson distribution amplitude, and $x_{3}$ the momentum fraction carried by the $\bar{c}$ quark. The effective Wilson coefficients including the $O\left(\alpha_{s}\right)$ vertex corrections are given by

$$
\begin{align*}
& a_{2}(x, \mu)=C_{1}(\mu)+\frac{C_{2}(\mu)}{N_{c}}\left[1+\frac{\alpha_{s}(\mu)}{4 \pi} C_{F}\left(-18+12 \ln \frac{m_{b}}{\mu}+f_{I}(x)\right)\right]  \tag{6}\\
& a_{3}(x, \mu)=C_{3}(\mu)+C_{9}(\mu)+\frac{1}{N_{c}}\left[C_{4}(\mu)+C_{10}(\mu)\right]\left[1+\frac{\alpha_{s}(\mu)}{4 \pi} C_{F}\left(-18+12 \ln \frac{m_{b}}{\mu}+f_{I}(x)\right)\right] \\
& a_{5}(x, \mu)=C_{5}(\mu)+C_{7}(\mu)+\frac{1}{N_{c}}\left[C_{6}(\mu)+C_{8}(\mu)\right]\left[1+\frac{\alpha_{s}(\mu)}{4 \pi} C_{F}\left(6-12 \ln \frac{m_{b}}{\mu}-f_{I}(x)\right)\right]
\end{align*}
$$

where the explicit expression for the function $f_{I}$ can be found in [15, 22],

$$
\begin{equation*}
f_{I}(x)=\frac{3(1-2 x)}{1-x} \ln x-3 \pi i+3 \ln \left(1-r_{3}^{2}\right)+\frac{2 r_{3}^{2}(1-x)}{1-r_{3}^{2} x} \tag{7}
\end{equation*}
$$

with the ratio $r_{3}=m_{J / \psi} / m_{B}, m_{B}$ being the $B$ meson mass. Compared to [8], the hard scale $t$ for the strong coupling constant and for the Wilson coefficients has been chosen as the larger internal momentum transfers between $\left(P_{2}+x_{3} P_{3}\right)^{2} \approx x_{3}\left(m_{B}^{2}-m_{J / \psi}^{2}\right)$ and $\left(P_{2}+\overline{x_{3}} P_{3}\right)^{2} \approx \overline{x_{3}}\left(m_{B}^{2}-m_{J / \psi}^{2}\right)$ with the notation $\overline{x_{3}}=1-x_{3}$, i.e.,

$$
\begin{equation*}
t=\max \left(\sqrt{x_{3}\left(m_{B}^{2}-m_{J / \psi}^{2}\right)}, \sqrt{\overline{x_{3}}\left(m_{B}^{2}-m_{J / \psi}^{2}\right)}\right) \tag{8}
\end{equation*}
$$

This choice is more consistent with the derivation of the PQCD formalism [23] and follows the well-known Brodsky-Lepage-Mackenzie procedure [24].

The leading-power $O\left(\alpha_{s}\right)$ nonfactorizable spectator amplitudes in PQCD are quoted from [8]:

$$
\begin{equation*}
\mathcal{A}_{J / \psi K^{0}}^{(c) n f}=\mathcal{M}\left(a_{2}^{\prime}\right), \quad \mathcal{A}_{J / \psi K^{0}}^{(t) n f}=\mathcal{M}\left(a_{3}^{\prime}-a_{5}^{\prime}\right) \tag{9}
\end{equation*}
$$

where the effective Wilson coefficients are defined by

$$
\begin{equation*}
a_{2}^{\prime}(\mu)=\frac{C_{2}(\mu)}{N_{c}}, \quad a_{3}^{\prime}(\mu)=\frac{1}{N_{c}}\left[C_{4}(\mu)+C_{10}(\mu)\right], \quad a_{5}^{\prime}(\mu)=\frac{1}{N_{c}}\left[C_{6}(\mu)+C_{8}(\mu)\right] \tag{10}
\end{equation*}
$$

The amplitude $\mathcal{M}$ is written as

$$
\begin{align*}
\mathcal{M}\left(a_{i}^{\prime}\right)=16 & \pi C_{F} \sqrt{2 N_{c}} \int_{0}^{1} d x_{1} d x_{2} d x_{3} \int_{0}^{\infty} b_{1} d b_{1} \phi_{B}\left(x_{1}, b_{1}\right) \\
\times\{ & {\left[\left(1-2 r_{3}^{2}\right) \overline{x_{3}} \phi_{K}^{A}\left(\overline{x_{2}}\right) \Psi^{L}\left(x_{3}\right)+\frac{1}{2} r_{3}^{2} \phi_{K}^{A}\left(\overline{x_{2}}\right) \Psi^{t}\left(x_{3}\right)\right.} \\
& \left.\quad-r_{2}\left(1-r_{3}^{2}\right) x_{2} \phi_{K}^{P}\left(\overline{x_{2}}\right) \Psi^{L}\left(x_{3}\right)+r_{2}\left(2 r_{3}^{2} \overline{x_{3}}+\left(1-r_{3}^{2}\right) x_{2}\right) \phi_{K}^{T}\left(\overline{x_{2}}\right) \Psi^{L}\left(x_{3}\right)\right] \\
\times & \alpha_{s}\left(t_{d}^{(1)}\right) a_{i}^{\prime}\left(t_{d}^{(1)}\right) S\left(t_{d}^{(1)}\right) h_{d}^{(1)}\left(x_{1}, x_{2}, x_{3}, b_{1}\right) \\
- & {\left[\left(x_{3}+\left(1-2 r_{3}^{2}\right) x_{2}\right) \phi_{K}^{A}\left(\overline{x_{2}}\right) \Psi^{L}\left(x_{3}\right)+r_{3}^{2}\left(2 r_{2} \phi_{K}^{T}\left(\overline{x_{2}}\right)-\frac{1}{2} \phi_{K}^{A}\left(\overline{x_{2}}\right)\right) \Psi^{t}\left(x_{3}\right)\right.} \\
& \left.\quad-r_{2}\left(1-r_{3}^{2}\right) x_{2} \phi_{K}^{P}\left(\overline{x_{2}}\right) \Psi^{L}\left(x_{3}\right)-r_{2}\left(2 r_{3}^{2} x_{3}+\left(1-r_{3}^{2}\right) x_{2}\right) \phi_{K}^{T}\left(\overline{x_{2}}\right) \Psi^{L}\left(x_{3}\right)\right] \\
\times & \left.\alpha_{s}\left(t_{d}^{(2)}\right) a_{i}^{\prime}\left(t_{d}^{(2)}\right) S\left(t_{d}^{(2)}\right) h_{d}^{(2)}\left(x_{1}, x_{2}, x_{3}, b_{1}\right)\right\}, \tag{11}
\end{align*}
$$

with the color factor $C_{F}=4 / 3$, the mass ratio $r_{2}=m_{0 K} / m_{B}, m_{0 K}$ being the chiral enhancement scale associated with the kaon, the impact parameter $b_{1}$ conjugate to the transverse momentum of the spectator quark, the two-parton twist-3 $J / \psi$ meson distribution amplitude $\Psi^{t}$, and the notation $\overline{x_{2}}=1-x_{2}$. The explicit expressions of the hard functions $h_{d}^{(1,2)}$, the hard scales $t_{d}^{(1,2)}$, and the Sudakov factor $S$ are referred to [8].

The penguin correction from the $u$-quark loop, which is of next-to-leading order in $\alpha_{s}$, can be expressed in terms of the effective Hamiltonian [5],

$$
\begin{align*}
\mathcal{H}_{\mathrm{eff}}^{(u)}= & -\frac{G_{F}}{\sqrt{2}} V_{u s}^{*} V_{u b}\left[\frac{\alpha}{3 \pi} e_{u} e_{c}\left(N_{c} C_{1}+C_{2}\right)(\bar{c} c)_{V}(\bar{s} b)_{V-A}+\frac{\alpha_{s}}{3 \pi} C_{2}\left(\bar{c} T^{a} c\right)_{V}\left(\bar{s} T^{a} b\right)_{V-A}\right] \\
& \times\left(\frac{2}{3}-\ln \frac{l^{2}}{\mu^{2}}+i \pi\right) . \tag{12}
\end{align*}
$$

The first (second) operator arises from the process, where the $c$-quark pair is produced through the photon (gluon) emission from the $u$-quark loop. $l^{2}$ is the invariant mass squared of the photon or gluon. Notice the constant $2 / 3$ different from $5 / 3$ in [迥], since we have considered the Fierz transformation of the four-fermion operators with the anticommuting $\gamma_{5}$ in $D$ dimensions under the NDR scheme [25]. We have also included the appropriate color factor $N_{c}$ and the charge factor $e_{u} e_{c}=4 / 9$. It is easy to observe that the first operator can be decomposed into $O_{7}$ and $O_{9}$, and the second operator into $O_{3-6}$.

The leading contribution from the first operator is a factorizable amplitude,

$$
\begin{equation*}
\mathcal{A}_{J / \psi K^{0}}^{(u 1) f}=2 \sqrt{2 N_{c}} \int_{0}^{1} d x_{3} \Psi^{L}\left(x_{3}\right) a_{\mathrm{ew}}\left(x_{3}, t\right) F_{+}^{\mathrm{BK}}\left(m_{J / \psi}^{2}\right), \tag{13}
\end{equation*}
$$

with the effective Wilson coefficient,

$$
\begin{equation*}
a_{\mathrm{ew}}(x, \mu)=-\frac{\alpha}{3 \pi} e_{u} e_{c}\left[N_{c} C_{1}(\mu)+C_{2}(\mu)\right]\left(\frac{2}{3}-\ln \frac{m_{J / \psi}^{2}}{\mu^{2}}+i \pi\right) \tag{14}
\end{equation*}
$$

where $l^{2}$ has been set to $m_{J / \psi}^{2}$. We have examined the average hard scale for the factorizable contribution, which is slightly above 3 GeV (the average hard scale for the nonfactorizable
contribution is between 1 and 1.5 GeV ). Simply comparing eq. (14) with eq. (7) for $\mu \approx$ 3 GeV , we find that the magnitude of eq. (13) is less than $5 \%$ of $\left|\mathcal{A}_{J / \psi K^{0}}^{(t) f}\right|$. Including the higher-order terms, such as the nonfactorizable amplitude, the effect from the first operator is expected to be around $5 \%$, and can be safely dropped.

For the second operator in eq. (12) to contribute, an additional gluon must attach the $c$-quark pair in the color-octet state. If it is a soft gluon, the corresponding nonperturbative input is the three-parton $J / \psi$ meson distribution amplitude, defined via the nonlocal matrix element [26],

$$
\begin{align*}
\left\langle J / \psi\left(P, \epsilon^{*(\lambda)}\right)\right| \bar{c}(-z) g_{s} G_{\mu \nu}(v z) & \gamma_{\alpha} c(z)|0\rangle=i f_{J / \psi} m_{J / \psi} P_{\alpha}\left[P_{\nu} \epsilon_{\perp \mu}^{*(\lambda)}-\epsilon_{\perp \nu}^{*(\lambda)} P_{\mu}\right] \tilde{V}(v, P \cdot z) \\
& +i f_{J / \psi} m_{J / \psi}^{3} \frac{\epsilon^{*(\lambda)} \cdot z}{P \cdot z}\left[P_{\mu} g_{\alpha \nu}^{\perp}-P_{\nu} g_{\alpha \mu}^{\perp}\right] \tilde{\Phi}(v, P \cdot z) \\
& +i f_{J / \psi} m_{J / \psi}^{3} \frac{\epsilon^{*(\lambda)} \cdot z}{(P \cdot z)^{2}} P_{\alpha}\left[P_{\mu} z_{\nu}-P_{\nu} z_{\mu}\right] \tilde{\Psi}(v, P \cdot z) \tag{15}
\end{align*}
$$

with the polarization vectors $\epsilon^{(\lambda)}$ of the $J / \psi$ meson, the gluon field strength tensor $G_{\mu \nu}$, and the projector $g_{\mu \nu}^{\perp}=g_{\mu \nu}-\left(P_{\mu} z_{\nu}+P_{\nu} z_{\mu}\right) /(P \cdot z), z$ being the coordinate of the $\bar{c}$ quark field. Because $m_{J / \psi} / m_{B}$ is not a small ratio, we extend our investigation to twist 4 . The twist-3 distribution amplitude $\tilde{V}$, involved only in decays into transversely polarized $J / \psi$ mesons, is irrelevant here. However, it could be a source of uncertainty for the extraction of $\sin \left(2 \phi_{1}\right)$ from the $B \rightarrow J / \psi K^{*}$ decays. The twist- 4 distribution amplitudes $\Phi\left(x_{c}, x_{\bar{c}}, x_{g}\right)$ and $\Psi\left(x_{c}, x_{\bar{c}}, x_{g}\right)$, from the Fourier transformation of $\tilde{\Phi}$ and $\tilde{\Psi}$, respectively, are antisymmetric under the exchange of the momentum fraction $x_{c}$ of the $c$ quark and $x_{\bar{c}}$ of the $\bar{c}$ quark [26], $x_{g}$ being the soft gluon momentum fraction. Because the hard kernel associated with the $u$-quark loop is symmetric under the exchange of $x_{c}$ and $x_{\bar{c}}$, its convolution with $\Phi$ and $\Psi$ diminishes. That is, the contribution from the second operator through higher-power terms is suppressed.

If the additional gluon is hard, the resultant contribution, being of $O\left(\alpha_{s}^{2}\right)$ (the gluon emitted by the $u$-quark loop is also hard), is beyond the scope of this paper. Nevertheless, we shall have a closer look at this piece. When the additional gluon is emitted by the spectator quark, the corresponding amplitude has the formula the same as $\mathcal{A}_{J / \psi K^{0}}^{(t) n f}$ but with the replacement,

$$
\begin{equation*}
a_{3,5}^{\prime} \rightarrow \frac{\alpha_{s}}{12 N_{c} \pi} C_{2}\left(\frac{2}{3}-\ln \frac{l^{2}}{\mu^{2}}+i \pi\right) \tag{16}
\end{equation*}
$$

Since the argument of $\mathcal{A}_{J / \psi K^{0}}^{(t) n f}$ depends on the difference of $a_{3}^{\prime}$ and $a_{5}^{\prime}$ as shown in eq. (9), the above corrections cancel each other exactly. When the additional gluon is emitted by the $b$ quark, the $s$ quark, or the $u$-quark loop, the corresponding amplitude involves a two-loop calculation, for which there is no simple estimation. The two-loop vertex corrections from various operators are expected to be finite, so their complete analysis for both $\mathcal{A}_{J / \psi K^{0}}^{(u)}$ and $\mathcal{A}_{J / \psi K^{0}}^{(t)}$ may be required in the future. Following the above analysis, we shall also ignore the $c$-quark-loop correction, which modifies only the prediction for the branching ratio slightly.

The choices of the $b$ quark mass, the $B$ meson wave function $\phi_{B}$, the $B$ meson lifetime, the twist-2 kaon distribution amplitude $\phi_{K}^{A}$, the two-parton twist-3 kaon distribution amplitudes $\phi_{K}^{P}$ and $\phi_{K}^{T}$, the $B$ meson and kaon decay constants, the chiral enhancement scale, and the CKM matrix elements, including their allowed ranges, are the same as in [27]. We take $m_{J / \psi}=3.097 \mathrm{GeV}$, and the $J / \psi$ meson distribution amplitudes [8, [28],

$$
\begin{align*}
\Psi^{L}(x) & =N^{L} \frac{f_{J / \psi}}{2 \sqrt{2 N_{c}}} x(1-x)\left[\frac{x(1-x)}{1-4 \alpha x(1-x)}\right]^{\alpha}, \\
\Psi^{t}(x) & =N^{t} \frac{f_{J / \psi}}{2 \sqrt{2 N_{c}}}(1-2 x)^{2}\left[\frac{x(1-x)}{1-4 \alpha x(1-x)}\right]^{\alpha}, \tag{17}
\end{align*}
$$

with the decay constant $f_{J / \psi}=405 \mathrm{MeV}$ [29]. The shape parameter will be varied between $\alpha=0.7 \pm 0.1$ to acquire the theoretical uncertainty from these nonperturbative inputs. It turns out that our results are not sensitive to the variation of $\alpha$. The normalization constants $N^{L, t}$ are determined by $\int d x \Psi^{L, t}(x)=f_{J / \psi} /\left(2 \sqrt{2 N_{c}}\right)$, giving the central values $N^{L}=9.58$ and $N^{t}=10.94$. The form factor $F_{+}^{\mathrm{BK}}\left(m_{J / \psi}^{2}\right)=0.62 \pm 0.09$ is quoted from the light-cone sum-rule calculation [30], to which $15 \%$ theoretical uncertainty has been assigned.

It is observed that $\mathcal{A}_{J / \psi K^{0}}^{(c) f}$ and $\mathcal{A}_{J / \psi K^{0}}^{(c) n f}$ are of the same order, confirming the importance of the nonfactorizable contribution in color-suppressed $B$ meson decays. Moreover, their real parts cancel, such that $\mathcal{A}_{J / \psi K^{0}}^{(c)}=\mathcal{A}_{J / \psi K^{0}}^{(c) f}+\mathcal{A}_{J / \psi K^{0}}^{(c) n}$ is almost imaginary. The amplitudes $\mathcal{A}_{J / \psi K^{0}}^{(c, t)}$ lead to the branching ratio, and $\Delta S_{J / \psi K_{S}}$ and $A_{J / \psi K_{S}}$ from the correction to the decay,

$$
\begin{align*}
B\left(B^{0} \rightarrow J / \psi K^{0}\right) & =\left(6.6_{-2.3(-2.3)}^{+3.7(+3.7)}\right) \times 10^{-4} \\
\Delta S_{J / \psi K_{S}}^{\text {decay }} & =\left(7.2_{-3.4(-1.1)}^{+2.4(+1.2)}\right) \times 10^{-4} \\
A_{J / \psi K_{S}}^{\text {decay }} & =-\left(16.7_{-8.7(-4.1)}^{+6.6(+3.8)}\right) \times 10^{-4} \tag{18}
\end{align*}
$$

where the errors in the parentheses arise only from the variation of the hadronic parameters. The result for the $B^{0} \rightarrow J / \psi K^{0}$ branching ratio is in agreement with the data $B\left(B^{0} \rightarrow\right.$ $\left.J / \psi K^{0}\right)=(8.72 \pm 0.33) \times 10^{-4}$ [31]. A form factor $F^{\mathrm{BK}}\left(m_{J / \psi}^{2}\right)$ increased by $15 \%$ and larger Gegenbauer coefficients in the kaon distribution amplitudes can easily account for the central value of the data. Our $\Delta S_{J / \psi K_{S}}^{\text {decay }}\left(A_{J / \psi K_{S}}^{\text {decay }}\right)$ from the penguin operators is about twice of the naive estimation from the $u$-quark-loop contribution [5] and has an opposite (the same) sign. Both $\Delta S_{J / \psi K_{S}}^{\text {deca }}$ and $A_{J / \psi K_{S}}^{\text {decay }}$ in eq. (18) indicate the $O\left(10^{-3}\right)$ penguin pollution in the $B \rightarrow J / \psi K_{S}$ decay, consistent with the conjecture made in [17].

We have checked that the correction to the mixing does not modify the denominator $1+\left|\lambda_{J / \psi K_{S}}\right|^{2}$ in the definitions of $S_{J / \psi K_{S}}$ and $A_{J / \psi K_{S}}$ in eq. (1). Hence, it is legitimate to simply add the values in eqs. (2) and (3) to eq. (18). We then derive $\Delta S_{J / \psi K_{S}}$ and $A_{J / \psi K_{S}}$ in the most complete analysis up to leading-power in $1 / m_{b}$ and to next-to-leading order in $\alpha_{s}$,

$$
\begin{align*}
\Delta S_{J / \psi K_{S}} & =\Delta S_{J / \psi K_{S}}^{\text {decay }}+\Delta S_{J / \psi K_{S}}^{\operatorname{mix}}=\left(9.3_{-4.6}^{+3.6}\right) \times 10^{-4} \\
A_{J / \psi K_{S}} & =A_{J / \psi K_{S}}^{\text {decay }}+A_{J / \psi K_{S}}^{\mathrm{mix}}=-\left(14.1_{-10.2}^{+8.1}\right) \times 10^{-4} \tag{19}
\end{align*}
$$

Including the CP violation from the $K-\bar{K}$ mixing [17], $\Delta S_{J / \psi K_{S}}$ and $A_{J / \psi K_{S}}$ remain $O\left(10^{-3}\right)$. Equation (19) will provide an important standard-model reference for verifying new physics from the $B^{0} \rightarrow J / \psi K_{S}$ data in the future. The estimate of $\Delta S_{J / \psi K_{S}} \sim$ $O\left(10^{-4}\right)$ in [5] implies that they have overlooked the more crucial contribution from the penguin operators. Our observation of the direct CP asymmetry supports the claim in [4] that $A_{\mathrm{CP}}(B \rightarrow J / \psi K)$ observed at $1 \%$ or higher would indicate new physics definitely.

## Acknowledgments

We thank W.S. Hou for a useful discussion. This work was supported by the National Science Council of R.O.C. under Grant No. NSC-95-2112-M-050-MY3, by the National Center for Theoretical Sciences, and by the U.S. Department of Energy under Grant No. DE-FG02-90ER40542. HNL thanks the Yukawa Institute for Theoretical Physics, Kyoto University, for her hospitality during his visit.

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